سینتیک و طرح راکتور - دکتر لطف اللهی

مرور حساب دیفرانسیل و انتگرال

$$y = x^2 \longrightarrow y' = \frac{dy}{dx} = 2x$$

$$y = x^n \longrightarrow y' = \frac{dy}{dx} = \text{شیب منحنی} = n x^{n-1}$$

chain rule $\longrightarrow dy = n x^{n-1} dx$

$$y = \int dy = \int n x^{n-1} dx = \cancel{n} \times \frac{1}{\cancel{(n-1+1)}} x^{n-1+1}$$

اگر انتگرال نامعین باشد $\quad y = x^n$

در انتگرال معین $\quad \int_{x_1}^{x_2} n x^{n-1} dx = x^n \Big]_{x_1}^{x_2} = x_2^n - x_1^n$

$$y = \int x^n dx = \frac{1}{n+1} x^{n+1} \qquad n \neq 1$$

$$y = \int x^{-1} dx = \int \frac{dx}{x} = \ln x \qquad n = 1$$

$$y = \int x^{-2} dx = \int \frac{dx}{x^2} = \frac{1}{-2+1} x^{-2+1} = -x^{-1} = \frac{-1}{x}$$

$$y = \int x^{-3} dx = \int \frac{dx}{x^3} = \frac{1}{-3+1} x^{-3+1} = \frac{-x^{-2}}{2}$$

$$y = \int \frac{dx}{x+2} = \int \frac{du}{u} = \ln u = \ln (x+2)$$

روش تغییر متغیر

1 $\quad x + 2 = u \longrightarrow dx = du$

$$y=\int \frac{dx}{(3x+2)^2}=\int \frac{\frac{du}{3}}{u^2}=\frac{1}{3}\times\frac{1}{-2+1}u^{-2+1}$$

$$y=-\frac{1}{3}u^{-1}$$

روش تغییر متغیر $3x+2=u \rightarrow 3dx=du$

$$y=-\frac{1}{3}(3x+2)^{-1}$$

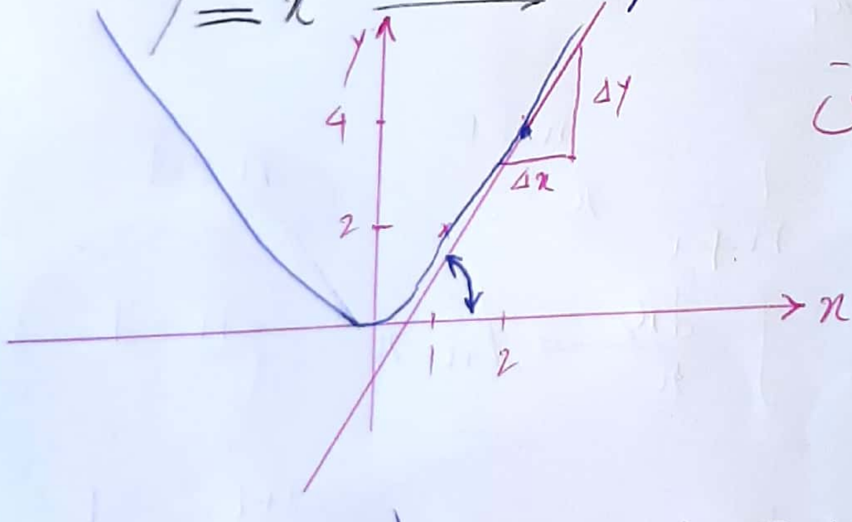
$$y=\int \frac{dx}{(ax+b)^n}=\frac{1}{a}\frac{1}{-n+1}(ax+b)^{-n+1} \qquad n\neq 1$$

$$y=\int \frac{dx}{ax+b}=\frac{1}{a}\ln(ax+b)$$

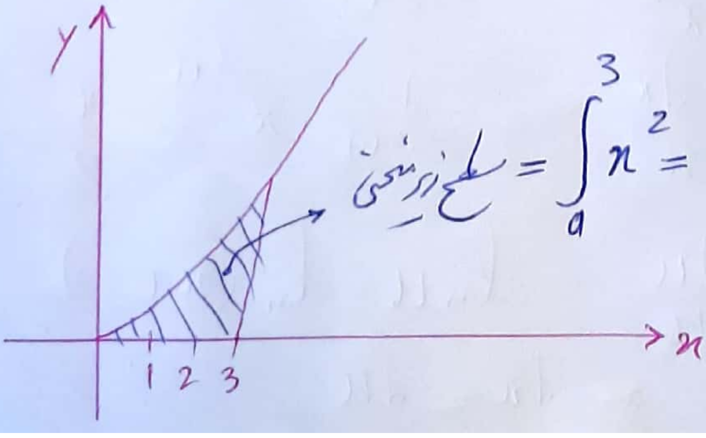
مفهوم مشتق و انتگرال

$y=x^2 \longrightarrow y'=2x=$ شیب خط مماس در نقطه x

مفهوم مشتق



$x=2$ شیب در نقطه $=$ slope $=2\times x=2\times 2=4=\tan(\alpha)$



سطح زیر منحنی $=\int_0^3 x^2=\frac{1}{3}x^3\Big|_0^3=\frac{1}{3}(3^3-0^3)=\frac{27}{3}=9$

مفهوم انتگرال

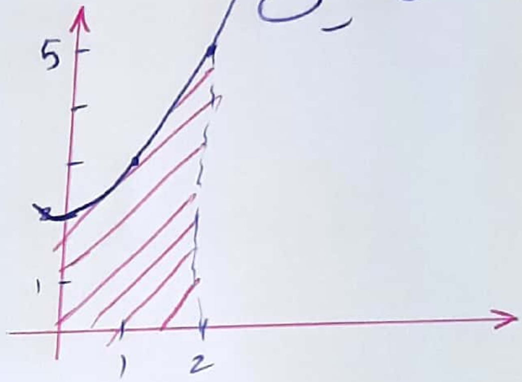
روش های تعیین انتگرال:
سه روش وجود دارد.

$$f(x) = x^2 + 1$$

1) روش تحلیلی:

$$I = \int_0^2 (x^2+1)\,dx = \frac{1}{3}x^3 + x \Big|_0^2 = \frac{8}{3} + 2 - 0 = \frac{14}{3}$$

روش دقیق



2) روش گرافیکی: $I =$ مساحت سطح زیر منحنی $=$ با استفاده از کاغذ میلیمتری

3) روش عددی:

$$I = \int_{x_0}^{x_n} (x^2+1)\,dx = \frac{h}{2}\left(f(x_0) + 2f(x_1) + 2f(x_2) + \dots f(x_n)\right)$$

روش ذوزنقه

$h = \Delta x = 0.5$

$x_0 = 0 , \quad x_n = 3$

$$I = \frac{0.5}{2}\Big[(0+1) + 2(0.5^2+1) + 2(1^2+1) + 2(1.5^2+1) + 2(2^2+1) + 2(2.5^2+1) + (3^2+1)\Big] =$$

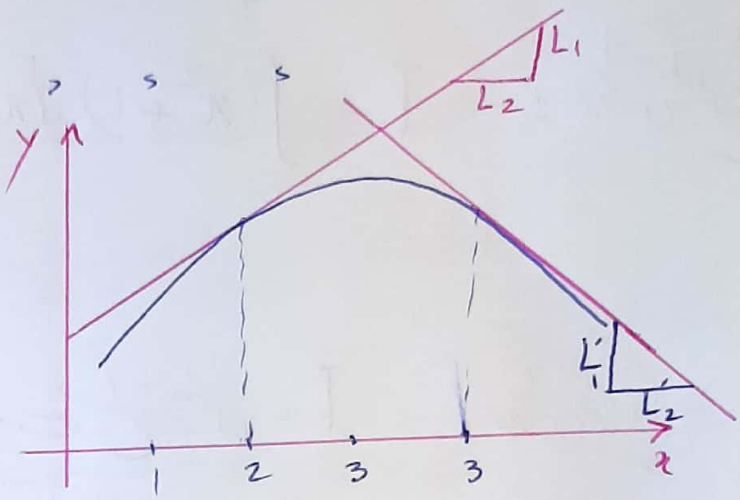
$$y = a^x = e^{x \ln a} = (e^{\ln a})^x$$

$$y' = \ln a \times a^x = \ln a \cdot e^{x \ln a}$$

مشتق:

$$\left.\frac{d f(x)}{dx}\right|_{x=2} = \frac{L_1}{L_2} = \text{شیب خط مماس بر منحنی}$$

$$\left.\frac{d f(x)}{dx}\right|_{x=4} = \frac{-L_1'}{L_2'} =$$



انتگرال:

روش تحلیلی:

$$I = \int_2^4 \frac{dx}{(1-x)^2} = \int_2^4 (1-x)^{-2} dx = \frac{1}{-2+1}(-1)(1-x)^{-2+1}$$

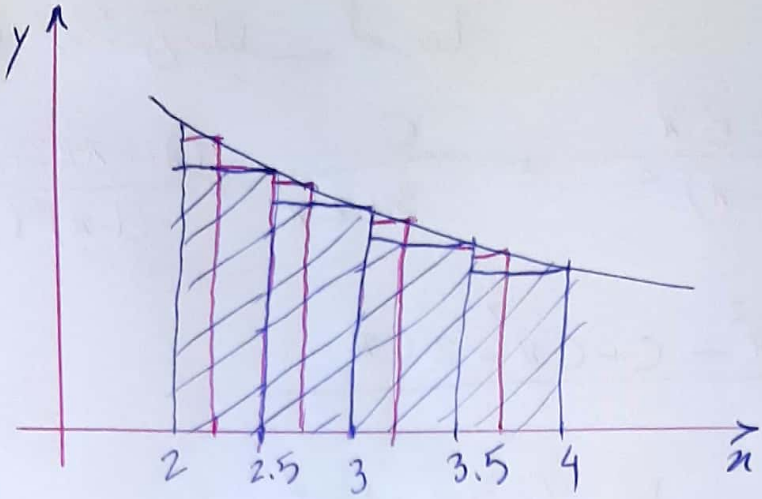
$$= (1-x)^{-1}\Big|_2^4 = \frac{1}{1-4} - \frac{1}{1-2} = \frac{-1}{3} + 1 = 2/3$$

روش عددی:

x	2	2.5	3	3.5	4
$f(x)=\frac{1}{(1-x)^2}$	1	$\frac{1}{(-1.5)^2}$	$\frac{1}{(1-3)^2}$	$\frac{1}{(1-3.5)^2}$	$\frac{1}{(1-4)^2}$
	f_0	f_1	f_2	f_3	f_4

$$I = \int \frac{dx}{(1-x)^2} = \frac{h}{2}\left(f_0 + 2f_1 + 2f_2 + 2f_3 + f_4\right)$$

$$= \frac{0.5}{2}\left(1 + 2\times\frac{1}{(1.5)^2} + 2\times\frac{1}{4} + 2\times\frac{1}{2.5^2} + \frac{1}{4}\right)$$



روش ذوزنقه در انتگرال عددی = سطح و مساحت مستطیل ها

$$* \quad I=\int \frac{2+3x}{x^2}dx=\int \frac{2\,dx}{x^2}+\int \frac{3x\,dx}{x^2}$$

$$I=2\,\frac{1}{(-2+1)}\,x^{-1}+3\ln x$$

$$*= \quad I=\int \frac{dx}{(1-x)(3-2x)}$$

روش تفکیک کسرها:

$$\frac{1+0\times x}{(1-x)(3-2x)}=\frac{A}{(1-x)}+\frac{B}{3-2x}=\frac{A(3-2x)+B(1-x)}{(1-x)(3-2x)}$$

$$=\frac{3A-2Ax+B-Bx}{(1-x)(3-2x)}=\frac{(3A+B)+x(2A-B)}{(1-x)(3-2x)}$$

$$\begin{cases} 3A+B=1 \longrightarrow 3A+2A=1 \longrightarrow A=\frac{1}{5} \\ 2A-B=0 \longrightarrow B=2A \longrightarrow B=\frac{2}{5} \end{cases}$$

$$\int\frac{dx}{(1-x)(3-2x)}=\int\frac{\frac{1}{5}dx}{1-x}+\int\frac{2/5\,dx}{3-2x}=-\frac{1}{5}\ln(1-x)+\frac{2}{5}\times\left(\frac{-1}{2}\right)\ln(3-2x)$$

۴ $I=\int \frac{dx}{(1-x)^2(3+2x)}$ روش تفکیک کسرها $(1+x^2-2x)$

$$\frac{1+0\times x+0\times x^2}{(1-x)^2(3+2x)}=\frac{A+Bx}{(1-x)^2}+\frac{C}{3+2x}=\frac{(A+Bx)(3+2x)+C(1-x)^2}{(1-x)^2(3+2x)}$$

$$=\frac{3A+3Bx+2Ax+2Bx^2+C+Cx^2-2Cx}{(1-x)^2(3+2x)}$$

$$=\frac{(3A+C)+(2A+3B-2C)x+(2B+C)x^2}{(1-x)^2(3+2x)}$$

$$\begin{cases}3A+C=1\\ 2A+3B-2C=0\\ 2B+C=0 \longrightarrow C=-2B \quad (2)\end{cases} \qquad \begin{cases}3A-2B=1 \quad (1)\\ 2A+3B-2(-2B)=0 \rightarrow 2A+7B=0\end{cases}$$

$$A=-\frac{7}{2}B$$

$$(1) \rightarrow 3\left(-\frac{7}{2}B\right)-2B=1 \rightarrow B=\frac{1}{-2-\frac{21}{2}}=\frac{-2}{25}$$

$$A=-\frac{7}{2}B=-\frac{7}{\cancel{2}}\times\frac{-\cancel{2}}{25}=\frac{7}{25}$$

$$(2) \rightarrow C=-2B=\frac{4}{25}$$

$$I=\int\frac{\frac{7}{25}-\frac{2}{25}x}{(1-x)^2}dx+\int\frac{\frac{4}{25}dx}{3+2x}=-\frac{7}{25}\times\frac{1}{-2+1}\frac{(-1)}{(1-x)}$$

$$-\frac{2}{25}\underbrace{\int\frac{x\,dx}{(1-x)^2}}+\frac{4}{25}\times\frac{1}{2}\ln(3+2x)=\frac{\frac{7}{25}}{1-x}-\frac{2}{25}\left(\frac{1}{1-x}-\ln(1-x)\right)+\frac{2}{25}\ln(3+2x)$$

انتگرال گیری با روش تغییر متغیر $\longrightarrow$ $1-x=u \longrightarrow x=1-u$, $-dx=du$

$$\int\frac{x\,dx}{(1-x)^2}=\int\frac{-(1-u)\,du}{u^2}=\int\frac{-du}{u^2}+\int\frac{u\,du}{u^2}=-\frac{1}{-2+1}u^{-1}$$

$$(6)\quad +\ln u=\frac{1}{1-x}+\ln(1-x)=\frac{1}{u}+\ln u$$